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ON THE UNIFORMLY STARLIKENSS OF THE EXPONENTIAL FUNCTION

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Abstract: **We introduce and study a new subclass of uniformly starlike functions by the means of the convolution involving the exponential function in the unit disk.**

Keywords: **Uniformly Starlike functions; Hadamard product; Linear operator; Exponential function.**

I. INTRODUCTION

Let A be the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

(1)

Which are analytic and univalent in the open disk $U = \{z: |z| < 1\}$

Also denote by T the subclass of A consisting of functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \, , a_n \geq 0$$

(2)

A function $f \in A$ is said to be in the class of uniformly convex functions of order α, denoted by $UCV(\alpha)$ if

And is said to be in a corresponding subclass of $UCV(\alpha)$ denoted by $S_p(\alpha)$ if

$$\Re\left\{\frac{zf'(z)}{f(z)} - \alpha\right\} \geq \beta \left|\frac{zf'(z)}{f(z)} - 1\right|,$$

$-1 \leq \alpha \leq 1$ and $z \in U$ The class of uniformly convex and uniformly starlike functions has been studied by Goodman, see [3,4] and Ma and Minda [6]. If f of the form (1) and $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$,are two functions in A, Then the Hadamard product of f and g is denoted by $f * g$ and is given by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n$$

Definition 1 we consider the following linear operator

$$Jf(z) = f(z) * exp(z) = z + \sum_{n=2}^{\infty} \frac{1}{n!} a_n z^n$$

(3)

where, $f(z)$ ∈A, and has the form (1).

Now using the operator introduced in (3) we can define the following subclass of analytic function, $J * f(z)$

$$\Re\left\{\frac{z(Jf(z))'}{Jf(z)}-\alpha\right\} \geq \left|\frac{z(Jf(z))'}{Jf(z)}-1\right|, z \in U$$

Now let's write $JTf(z) = J * f(z) \cap T$

The origin of such classes is introduced and studied by various authors including [1], [8], and [9].

II. MAIN RESULTS

1. Characterization Property

Theorem 1.

A function f defined by (2) is in the class $JTf(z)$if and only if

$$\sum_{n=2}^{\infty} \frac{1}{n!} \cdot \frac{2n-1-\alpha}{1-\alpha} |a_n| \leq 1$$

Proof. It suffices to show that

$$\left|\frac{z(Jf(z))'}{Jf(z)}-1\right| \leq \Re\left\{\frac{z(Jf(z))'}{Jf(z)}-\alpha\right\}$$

and we have

$$\left|\frac{z(Jf(z))'}{Jf(z)}-1\right| \leq \Re\left\{\frac{z(Jf(z))'}{Jf(z)}-1\right\} + (1-\alpha)$$

that is

$$\left|\frac{z(Jf(z))'}{Jf(z)}-1\right| - \Re\left\{\frac{z(Jf(z))'}{Jf(z)}-1\right\} \leq 2\left|\frac{z(Jf(z))'}{Jf(z)}-1\right| \leq \frac{\sum_{n=2}^{\infty}(n-1)\, 1/n!\, |a_n|}{1-\sum_{n=2}^{\infty} 1/n!\, |a_n|}$$

The above expression is bounded by $(1-\alpha)$ and hence the assertion of the result

Now we want to show that $f \in JTf(z)$ satisfies (3)

if $f \in JTf(z)$ then (3) yields

$$\frac{1-\sum_{n=2}^{\infty} n\, /n!\, a_n z^{n-1}}{1-\sum_{n=2}^{\infty} 1/n!\, a_n z^{n-1}} - \alpha \geq \frac{1-\sum_{n=2}^{\infty}(n-1)\, 1/n!\, a_n z^{n-1}}{1-\sum_{n=2}^{\infty} 1/n!\, a_n z^{n-1}}$$

Letting $z \to 1$ along the real axis leads to the inequality

$$\sum_{n=2}^{\infty}(2n-1-\alpha)\, 1/n!\, a_n \leq 1-\alpha$$

Corollary 1. let a function f, defined by (2) belongs to the class $JTf(z)$then

$a_n \leq \frac{1}{n!} \cdot \frac{1-\alpha}{2n-1-\alpha}$, for $n \geq 2$

Theorem 2 Let the function f, defined by (2) be in the class $JTf(z)$then

$$|z| - |z|^2 \frac{1}{2(3-\alpha} \leq |Jf(z)| \leq |z| + |z|^2 \frac{1}{2(3-\alpha)}$$

$$1 - |z| \frac{1}{(3-\alpha)} \leq |(Jf(z))'| \leq 1 + |z| \frac{1}{(3-\alpha)}$$

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The bounds above are attained for the functions given by

$$f(z) = z - \frac{1}{4(3-\alpha)} z^2$$

Theorem 3. Let a function f, be defined by (1.2) and

$$g(z) = z - \sum_{n=2}^{\infty} b_n z^n$$

be in the class $JTf(z)$. then the function h, defined by

$$h(z) = (1-\beta)f(z) + \beta g(z) = z - \sum_{n=2}^{\infty} c_n z^n$$

Where $c_n = (1-\beta)a_n + \beta b_n$,and $0 \leq \beta \leq 1$,is also in the class $JTf(z)$

Now we define the following functions $f_j(z)$,$(j = 1,2,3, \ldots\ldots m)$

of the form

$$f_j(z) = z - \sum_{n=2}^{\infty} a_{n,j} z^n, a_{n,j} \geq 0, z \in U$$

(4)

Theorem 4 *(closure theorem).* Let the functions $f_j(z)$ $(j = 1,2,3, \ldots\ldots m)$ defined by (4), be in the class $Jf_j(z)(j = 1,2,3, \ldots\ldots m)$ respectively. Then the function $h(z)$ defined by

$$h(z) = z - \frac{1}{m} \sum_{n=2}^{\infty} \left(\sum_{j=1}^{m} a_{n,j} \right) z^n$$

is in the class $Jf_\xi(z)$where

$\xi = \underset{1 \leq j \leq m}{Max} \{\alpha_j\} with \ \ 0 \leq \alpha_j < 1$.

2. Results involving convolution

Theorem 5. for functions $f_j(z)$ $(j = 1,2)$ defined by (4) let $f_1(z) \in JTf_1(z)$and $f_2(z) \in JTf_2(z)$. then $f_1 * f_2 \in TR(\eta, \lambda, \gamma)$, where

$$\gamma \leq 1 - \frac{1}{(2n-1-\alpha)(2n-1-\beta)1/n! - (1-\alpha)(1-\beta)},$$

Theorem 6 Let the functions $f_j(z), (j = 1,2)$ defined by (4) be in the class $JTf(z)$Then $(f_1 * f_2)(z) \in JT\_\rho f(z)$

3. The Integral Transform

We define the integral transform

$$V_\mu(f)(z) = \int_0^1 \mu(t) \frac{f(tz)}{t} dt$$

Where $\mu(t)$ is a real valued, non-negative weight function normalized so that $\int_0^1 \mu(t)dt = 1$.

Special case of $\mu(t)$ is $\mu(t) = \frac{(c+1)^\delta}{\mu(\delta)} t^c \left(\log\frac{1}{t}\right)^{\delta-1}$, $c > -1, \delta \geq 0$

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Which gives the Komatu operator.

Theorem 7 Let $f \in JTf(z)$Then $V_\mu(f) \in JTf(z)$.

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Theorem 8. *(radius of starlikeness)* Let $f \in JTf(z)$then $V_\mu(f)$is starlike of order $0 \le \gamma < 1$ in $|z| \prec R_1$, where

$$R_1 = \min_n \left[\left(\frac{c+n}{c+1}\right)^\delta \cdot \frac{1-\gamma(2n-1-\alpha)}{(n-\gamma)(1-\alpha)} . \frac{1}{n!}\right]^{\frac{1}{n-1}}$$

Theorem 9. $f \in JTf(z)$then $V_\mu(f)$is convex of order $0 \le \gamma < 1$,in $|z| < R_2$,where

$$R_2 = \min_n \left[\left(\frac{c+n}{c+1}\right)^\delta \frac{(1-\gamma)(2n-1-\alpha)}{n(n-\gamma)(1-\alpha)} . \frac{1}{n!}\right]^{\frac{1}{n-1}}$$

III. CONCLUSION

By utilizing the Hadamard product, we introduced a new linear operator that involves the analytic functions on the unit disk and the exponential function. Consequently, we defined a new subclass of uniformly starlike functions. We investigated the characteristics of the new subclass and determined the sufficient conditions for the inclusion results. In addition, we expended the finding to involve Hadamard product. Finally, we presented a study on the integral transform and obtained various results.

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